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ENGINEERING MATHEMATICS 1 :

MATRICES

FOR MALAYSIAN POLYTECHNIC STUDENTS

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With the name of Allah, Most Gracious, Most Merciful, the First and the Foremost. Our deepest gratitude extends to Allah S.W.T who has given us patience, strength, determination and courage to carry out the writing of this **ENGINEERING MATHEMATICS 1: MATRICES FOR MALAYSIAN POLYTECHNIC STUDENTS** e-book.

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Thank you.

ABSTRACT

ENGINEERING MATHEMATICS 1: MATRICES FOR MALAYSIAN POLYTECHNIC STUDENTS

This book introduces students to the fundamentals of matrices, operations on matrices and how to solve simultaneous linear equations up to three variables by using matrices.

This book contains THREE (3) main sections, various examples of work and tutorials to enhance students learning abilities. The authors hope this book can provide useful resources to students and lecturers.

This book is the full property of Politeknik Kuala Terengganu which is used on the online / offline learning platform. The production of this book is also suitable for use by community college, pre-diploma and university students.

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INTRODUCTION

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- Introduction

1.1.1

- Characteristics of Matrices

1.1.2

- Types of Matrices

MATRICES

1.1 INTRODUCTION

In mathematics, a matrix (plural matrices) is widely used in engineering, science, physics and most other scientific fields. This section will cover the characteristics of elements of a matrix, the order of a matrix and types of matrices.

1.1.1 CHARACTERISTICS OF MATRICES

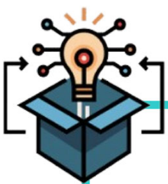
A matrix is a rectangular array of numbers, figures, or expressions, organized in **rows** and **columns**.

Example 1

$$A = \begin{pmatrix} 7 & 4 & -9 \\ 5 & 8 & 1 \\ 3 & -5 & 0 \end{pmatrix}$$

Rows

Columns

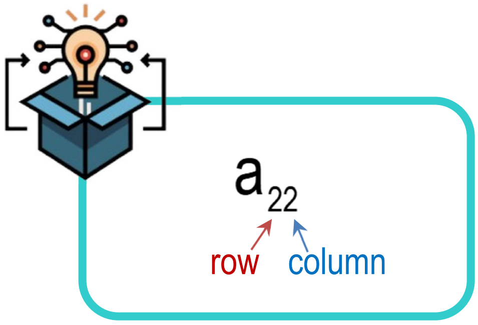


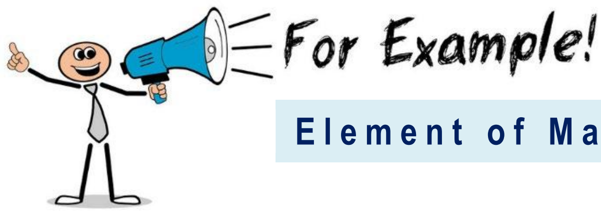
Matrices are usually symbolized using upper-case letters (such as A in the Example 1).

Elements of Matrices

The individual item (any numbers or symbols) in a matrix is called an element. Each element is identified by using its row number followed by its column number. Lower-case letters, with two subscript indices (e.g., a_{11}, a_{12}, a_{13}, a_{21}, a_{22} etc.), represent the element of matrix.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$





Element of Matrices

Example 2:

Given $A = \begin{pmatrix} 7 & 4 & -9 \\ 5 & 8 & 1 \\ 3 & -5 & 0 \end{pmatrix}$. Find A_{23} and A_{31}.

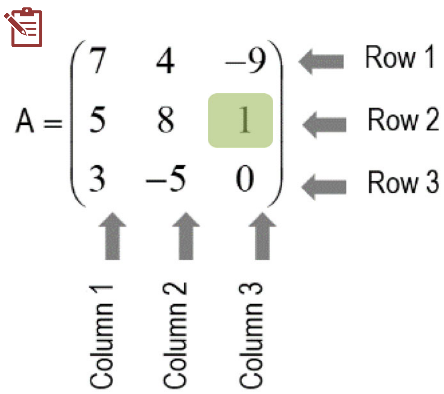
Solution

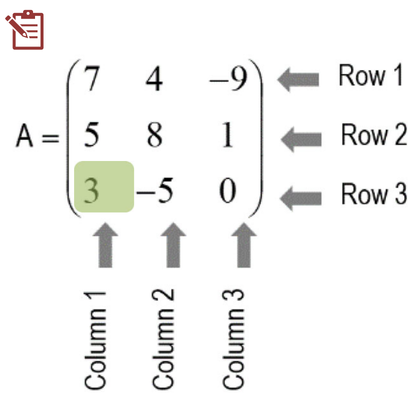
a) $A_{23} = \mathbf{1}$

row: 2 column: 3

b) $A_{31} = \mathbf{3}$

row: 3 column: 1





EXERCISES 1

MATH DRILLS!

Answer all the questions.

1. Given $B = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 5 & 0 \end{pmatrix}$

State the element for;

a) B_{11} = ________ b) B_{13} = ________ c) B_{22} = ________

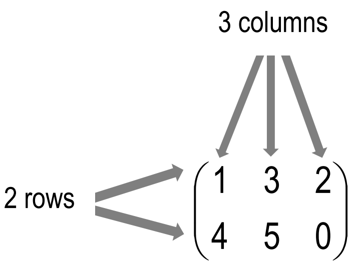
2. Given $C = \begin{pmatrix} 4 & 3 \\ 0 & 7 \\ -5 & 6 \end{pmatrix}$

State the element for;

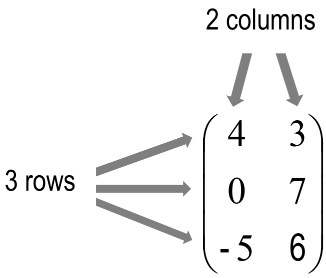
a) C_{12} = ________ b) B_{21} = ________ c) B_{32} = ________

Orders of Matrices

The order of a matrix is defined by the number of rows and columns that it contains. A matrix with ***m*** rows and ***n*** columns is called an ***m*** **×** ***n*** matrix.



2 × ***3*** matrix



3 × ***2*** matrix

Orders of Matrices

Example 3:

State the order of the following matrices.

a) $\begin{pmatrix} 1 & 2 & 5 & 0 \\ 7 & 3 & 8 & -1 \\ 4 & -7 & 9 & 6 \end{pmatrix}$ b) $\begin{pmatrix} 4 & 5 & 8 \end{pmatrix}$ c) $\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$

Solution

a) 3 × 4 matrix
b) 1 × 3 matrix
c) 3 × 1 matrix

EXERCISES 2

MATH DRILLS!

Answer all the questions.

State the order of the following matrices.

a) $\begin{pmatrix} 1 & 6 \end{pmatrix}$

b) $\begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 5 & 7 \\ 9 & 0 & -4 \\ 8 & 4 & 3 \end{pmatrix}$

d) $\begin{pmatrix} 3 & 7 \\ 1 & 6 \\ 9 & 2 \end{pmatrix}$

1.1.2 TYPES OF MATRICES

There are a variety of matrix such as square matrix, zero matrix, diagonal matrix, identity matrix and transpose matrix.

1. **Square Matrix** | A square matrix is a matrix with the same number of rows and columns.

$$A=\begin{pmatrix}1 & -3 & 5 & 0\\ 3 & 2 & 5 & 7\\ -6 & 0 & 3 & 8\\ 4 & 7 & 9 & 4\end{pmatrix} \qquad B=\begin{pmatrix}1 & -2 & 4\\ 6 & 7 & 0\\ 3 & 2 & 9\end{pmatrix} \qquad C=\begin{pmatrix}1 & 3\\ 4 & 1\end{pmatrix}$$

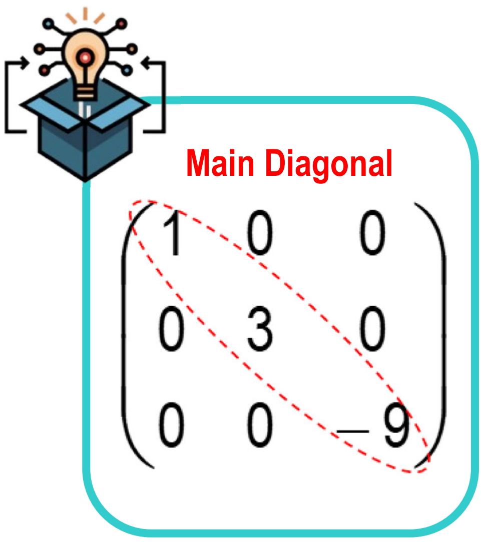
4×4 matrix $\quad$ 3×3 matrix $\quad$ 2×2 matrix

2. **Zero Matrix** | A matrix with zero as all its elements.

$$A=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

3. **Diagonal Matrix** | A diagonal matrix where all the elements is zero except the elements of main diagonal.

$$A=\begin{pmatrix}1 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & -9\end{pmatrix}$$

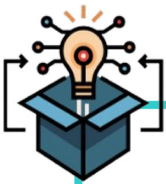


4 **Identity Matrix** | A square matrix in which all the elements on main diagonal are equals to 1 and all other elements are equal to zero.

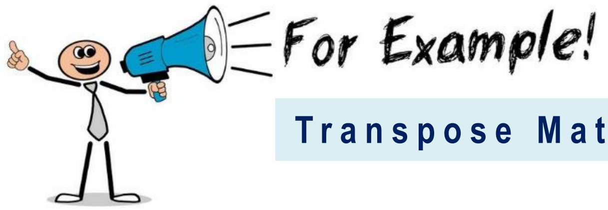
$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

5 **Transpose Matrix** | The transpose of an $m \times n$ matrix is obtained by interchanging the rows and columns of A.

$$A = \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ 8 & 5 \end{pmatrix}; \qquad A^T = \begin{pmatrix} 2 & 0 & 8 \\ 4 & -1 & 5 \end{pmatrix}$$



The transpose of matrix A is denoted by A^T



Transpose Matrix

<u>Example 4:</u>

Find the transpose of each of the following matrices.

a) $A = \begin{pmatrix} 3 & 5 & 9 \\ -1 & 2 & 8 \\ 4 & 0 & 7 \end{pmatrix}$

b) $B = \begin{pmatrix} 7 & -5 & 3 \\ 9 & 2 & 4 \end{pmatrix}$

Solution

a) $A^T = \begin{pmatrix} 3 & -1 & 4 \\ 5 & 2 & 0 \\ 9 & 8 & 7 \end{pmatrix}$

b) $B^T = \begin{pmatrix} 7 & 9 \\ -5 & 2 \\ 3 & 4 \end{pmatrix}$

02

OPERATIONS ON MATRICES

1.2 OPERATIONS ON MATRICES

Operations on matrices involve such as addition, subtraction and multiplication.

1.2.1 ADDITION AND SUBTRACTION OF MATRICES

The addition and subtraction **can be done on matrices of the same order**.

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \qquad B = \begin{pmatrix} j & k & l \\ m & n & o \\ p & q & r \end{pmatrix}$$

Addition

$$A + B = \begin{pmatrix} a+j & b+k & c+l \\ d+m & e+n & f+o \\ g+p & h+q & i+r \end{pmatrix}$$

To add two matrices: add the numbers in the same element positions.

Subtraction

$$A - B = \begin{pmatrix} a-j & b-k & c-l \\ d-m & e-n & f-o \\ g-p & h-q & i-r \end{pmatrix}$$

To subtract add two matrices: subtract the numbers in the same element positions.

For Example!

Addition and Subtraction of Matrices

Example 5:

The following matrices are given.

$$A=\begin{pmatrix}2 & -1 & 1\\ 2 & 4 & 1\end{pmatrix}\qquad B=\begin{pmatrix}0 & 1 & 1\\ 2 & -1 & 1\end{pmatrix}\qquad C=\begin{pmatrix}2 & 2 & 1\\ 3 & 5 & 4\\ 1 & 0 & -5\end{pmatrix}$$

Calculate :

i. $A+B$

ii. $A-B$

iii. $B+C$

iv. $B-C$

Solution

i. $$A+B=\begin{pmatrix}2 & -1 & 1\\ 2 & 4 & 1\end{pmatrix}+\begin{pmatrix}0 & 1 & 1\\ 2 & -1 & 1\end{pmatrix}$$
$$=\begin{pmatrix}2+0 & -1+1 & 1+1\\ 2+2 & 4+(-1) & 1+1\end{pmatrix}$$
$$=\begin{pmatrix}2 & 0 & 2\\ 4 & 3 & 2\end{pmatrix}$$

ii. $$A-B=\begin{pmatrix}2 & -1 & 1\\ 2 & 4 & 1\end{pmatrix}-\begin{pmatrix}0 & 1 & 1\\ 2 & -1 & 1\end{pmatrix}$$
$$=\begin{pmatrix}2-0 & -1-1 & 1-1\\ 2-2 & 4-(-1) & 1-1\end{pmatrix}$$
$$=\begin{pmatrix}2 & -2 & 0\\ 0 & 5 & 0\end{pmatrix}$$

iii. $B + C$

Cannot be done because the order of matrices are not the same.

iv. $B - C$

Cannot be done because the order of matrices are not the same.

Example 6:

Given $A = \begin{pmatrix} 3 & -1 & 4 & 10 \\ 1 & 3 & 3 & -2 \\ -7 & 1 & 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 5 & -8 & 0 \\ -4 & 1 & -3 & 0 \\ 1 & -3 & 0 & -1 \end{pmatrix}$. Determine $A - B$.

Solution

$$A - B = \begin{pmatrix} 3 & -1 & 4 & 10 \\ 1 & 3 & 3 & -2 \\ -7 & 1 & 5 & 3 \end{pmatrix} - \begin{pmatrix} -2 & 5 & -8 & 0 \\ -4 & 1 & -3 & 0 \\ 1 & -3 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3-(-2) & -1-5 & 4-(-8) & 10-0 \\ 1-(-4) & 3-1 & 3-(-3) & -2-0 \\ -7-1 & 1-(-3) & 5-0 & 3-(-1) \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -6 & 12 & 10 \\ 5 & 2 & 6 & -2 \\ -8 & 4 & 5 & 4 \end{pmatrix}$$

MATH DRILLS!

Answer all the questions.

EXERCISES 3

1. Given $A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 4 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 3 & 1 \\ 4 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 0 & -4 \\ 4 & -3 & -1 \end{pmatrix}$

a) $A+B$	b) $A+C$
c) $C-A$	d) $B-A^T$

2. Given $B = \begin{pmatrix} 2 & -1 \\ 4 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} -4 & 1 \\ 3 & 6 \end{pmatrix}$, calculate :

a) $B+C$	b) $B-C$
c) C^T-C	d) B^T-C^T

1.2.2 MULTIPLICATION OF MATRICES

Scalar Multiplication

Each element needs to be multiplied by the scalar value (a single number).

Scalar Multiplication

<u>Example 7:</u>

Given $A = \begin{pmatrix} 1 & 0 \\ 4 & -2 \\ 5 & 3 \end{pmatrix}$, find 2A.

Solution

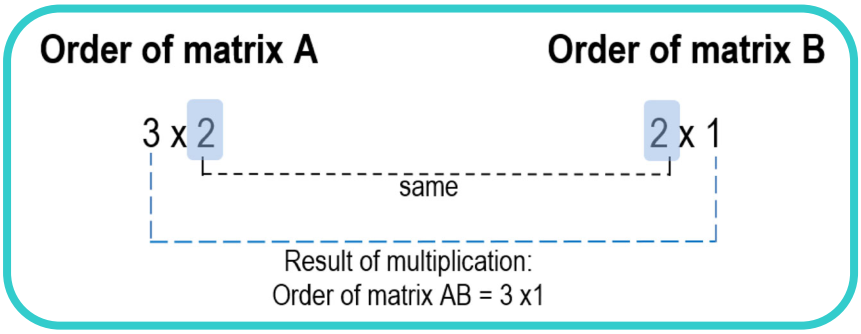
$$2A = 2\begin{pmatrix} 1 & 0 \\ 4 & -2 \\ 5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times 1 & 2\times 0 \\ 2\times 4 & 2\times(-2) \\ 2\times 5 & 2\times 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 8 & -4 \\ 10 & 6 \end{pmatrix}$$

Multiplication of Matrices

Two matrices can be multiplied if the number of columns in the first matrix is the same as the number of rows in the second matrix.



$$A = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}, \quad B = \begin{pmatrix} g \\ h \end{pmatrix}$$

First matrix — Second matrix

$$AB = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \begin{pmatrix} g \\ h \end{pmatrix}$$

Order of matrix: 3 x 2 — Order of matrix: 2 x 1

$$= \begin{pmatrix} ag + bh \\ cg + dh \\ eg + fh \end{pmatrix}$$

Order of matrix: 3 x 1 — Result multiplication

The multiplication of matrices is **not commutative**, which means in general that ***A.B ≠ B.A***

The multiplication of matrices is **associative**, which means that **(A.B).C = A.(B.C)**

For Example!

Multiplication of Two Matrices

Example 8:

Given $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -7 & 8 \\ 0 & 1 & -4 \\ 6 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ -3 & -1 \\ 4 & 5 \end{pmatrix}$, find

i) AB

ii) BA

Solution

i) AB

Remember:
Arrange the matrices as follows:

$$AB = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -7 & 8 \\ 0 & 1 & -4 \\ 6 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -3 & -1 \\ 4 & 5 \end{pmatrix}$$

First matrix (Left) — Order of matrix: 4 x 3

Second matrix (Right) — Order of matrix: 3 x 2

SAME: multiplication of matrix AB can be done

$$= \begin{pmatrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{pmatrix}$$

Result multiplication — Order of matrix: 4 x 2

The answer will be shown on the next page

$$AB = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -7 & 8 \\ 0 & 1 & -4 \\ 6 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -3 & -1 \\ 4 & 5 \end{pmatrix}$$

$$AB = \begin{pmatrix} (1\times 2)+(0\times -3)+(1\times 4) & (1\times 0)+(0\times -1)+(1\times 5) \\ (2\times 2)+(-7\times -3)+(8\times 4) & (2\times 0)+(-7\times -1)+(8\times 5) \\ (0\times 2)+(1\times -3)+(-4\times 4) & (0\times 0)+(1\times -1)+(-4\times 5) \\ (6\times 2)+(2\times -3)+(1\times 4) & (6\times 0)+(2\times -1)+(1\times 5) \end{pmatrix}$$

$$AB = \begin{pmatrix} 6 & 5 \\ 57 & 47 \\ -19 & -21 \\ 10 & 3 \end{pmatrix}$$

Tips: For better understanding, the multiplication of two matrices can be illustrated as table below. As sample, notice that dash type blue arrows involve multiplication with elements and dash type red arrows are addition operation in between that multiplication.

	2 -3 4	0 -1 5
1 0 1	$(1\times 2)+(0\times -3)+(1\times 4)$ $=6$	$(1\times 0)+(0\times -1)+(1\times 5)$ $=5$
2 -7 8	$(2\times 2)+(-7\times -3)+(8\times 4)$ $=57$	$(2\times 0)+(-7\times -1)+(8\times 5)$ $=47$
0 1 -4	$(0\times 2)+(1\times -3)+(-4\times 4)$ $=-19$	$(0\times 0)+(1\times -1)+(-4\times 5)$ $=-21$
6 2 1	$(6\times 2)+(2\times -3)+(1\times 4)$ $=10$	$(6\times 0)+(2\times -1)+(1\times 5)$ $=3$

i) BA

The multiplication of matrix B and A cannot be done because the numbers of columns in matrix B is not equal to the numbers of rows in matrix A.

Multiplication of Two Matrices

Example 9:

Given $C = \begin{pmatrix} 0 & -3 \\ 2 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, find CD.

Solution

$$CD = \begin{pmatrix} 0 & -3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$
$$= \begin{pmatrix} (0\times 2)+(-3\times 4) & (0\times 3)+(-3\times 5) \\ (2\times 2)+(1\times 4) & (2\times 3)+(1\times 5) \end{pmatrix}$$
$$= \begin{pmatrix} -12 & -15 \\ 8 & 11 \end{pmatrix}$$

Example 10:

Given $P = \begin{pmatrix} 3 & 1 & 4 \end{pmatrix}$ and $Q = \begin{pmatrix} 4 & 3 \\ 2 & 5 \\ 6 & 8 \end{pmatrix}$, find PQ.

Solution

$$PQ = \begin{pmatrix} 3 & 1 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 2 & 5 \\ 6 & 8 \end{pmatrix}$$
$$= \begin{pmatrix} (3\times 4)+(1\times 2)+(4\times 6) & (3\times 3)+(1\times 5)+(4\times 8) \end{pmatrix}$$
$$= \begin{pmatrix} 38 & 46 \end{pmatrix}$$

EXERCISES 4

MATH DRILLS!

Answer all the questions.

1. If $X = \begin{pmatrix} -1 & -3 \\ 4 & 0 \end{pmatrix}$ and $Y = \begin{pmatrix} 6 & 9 \\ 7 & 2 \end{pmatrix}$, find :

i) XY

ii) YX

2. Given $D = \begin{pmatrix} 3 & 5 & -3 \\ 0 & 8 & 4 \\ -1 & 7 & 2 \end{pmatrix}$ and $E = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 5 & 3 \end{pmatrix}$, find DE.

3. Given $P = \begin{pmatrix} 9 & 7 \\ 4 & 3 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & 5 \\ -7 & 6 \end{pmatrix}$, find $P^T Q$.

4. If $A = \begin{pmatrix} 1 & 2 \\ 5 & 3 \\ 7 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 5 & -3 \\ 0 & 6 & 4 \\ -1 & 7 & 2 \end{pmatrix}$, find BA.

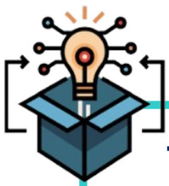
Determinant

The determinant is unique number from a square matrix. The determinant for matrix A is donated as det(A) or IAI.

The determinant 2x2 matrix is defined by:

$$\text{If } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$
$$= ad - bc$$



The determinant is used to find the inverse matrix that will be used to solve the system of linear equation.

Determinant 2x2 Matrix

Example 11:

$$\text{If } A = \begin{pmatrix} 2 & -1 \\ 0 & 5 \end{pmatrix}, \text{find } |A|.$$

Solution

$$|A| = \begin{vmatrix} 2 & -1 \\ 0 & 5 \end{vmatrix}$$
$$= (2 \times 5) - (-1 \times 0)$$
$$= 10 - (0)$$
$$= 10$$

For a 3x3 matrix, the determinants can be obtained using two different methods which are cofactor expansion and cross multiplication.

$$\text{If } A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

Cofactor expansion

$$\therefore |A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

The determinant is:

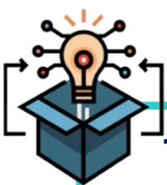
$$a(ei - fh) - b(di - fg) + c(dh - eg)$$

Cross multiplication

$$\therefore |A| = \left(\begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array}\right)$$

The determinant is:

$$((aei + bfg + cdh) - (gec + hfa + idb))$$



Total of multiplication for arrows down (blue arrows) minus total of multiplication for arrows up (red arrows).

Determinant 3x3 Matrix

Example 12:

If $A = \begin{pmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{pmatrix}$, find $|A|$.

Solution

By using cofactor expansion:

$$|A| = -3\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} - (-2)\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} + 2\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$$

$$= -3\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} + 2\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$$

$$= -3[(3x5) - (2x4)] + 2[(1x5) - (2x2)] + 2[(1x4) - (3x2)]$$

$$= -3(15 - 8) + 2(5 - 4) + 2(4 - 6)$$

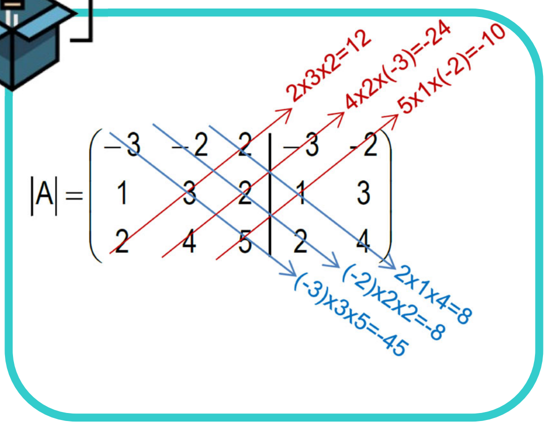
$$= -3(7) + 2(1) + 2(-2)$$

$$= -21 + 2 - 4$$

$$= -23$$

Or by using cross multiplication

$$\therefore |A| = \left(\begin{array}{ccc|cc} -3 & -2 & 2 & -3 & -2 \\ 1 & 3 & 2 & 1 & 3 \\ 2 & 4 & 5 & 2 & 4 \end{array}\right)$$



$$=[8+(-8)+(-45)]-[12+(-24)+(-10)]$$

$$= -45-(-22)$$

$$= -23$$

MATH DRILLS!

Answer all the questions.

EXERCISES 5

Find the determinant of matrix below.

a) $A = \begin{pmatrix} 2 & 5 \\ 4 & -1 \end{pmatrix}$	b) $Q = \begin{pmatrix} 7 & -1 \\ 2 & 3 \end{pmatrix}$
c) $P = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 3 & 0 \\ 2 & 1 & 2 \end{pmatrix}$	d) $B = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 0 & 3 \\ 1 & 3 & 4 \end{pmatrix}$

03

SOLVING SIMULTANEOUS LINEAR EQUATION UP TO THREE VARIABLES

1.3

- Solving Simultaneous Linear Equation up to Three Variables

1.3.1

- Inverse Matrix Method

1.3.2

- Cramer's Rule

1.3 Solving Simultaneous Linear Equation up to Three Variables

1.3.1 Inverse Matrix Method

The inverse of square matrix A, denoted by A^{-1} can be found by using the following equation, where adj(A) denotes the adjoint of matrix A and the determinant for matrix A is donated as det(A) or IAI. It can be calculated by the following method:

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

Step to find an inverse matrix.

1. Find the determinant of the matrix.
2. Find the minor of the matrix.
3. Find the cofactor of the matrix.
4. Find the adjoint of the matrix by transpose the cofactor.
5. Find the inverse using:

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

For 2x2 matrix:

MINOR

If A is a square matrix, the minor of the element is the determinant of the sub matrix formed by deleting the i-th row and j-th column. This number is often denoted M_{ij}.

$$\text{Let } A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\text{Therefore, Minor } A = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

Where :

$$M_{11} = a_{22} \quad M_{12} = a_{21}$$
$$M_{21} = a_{12} \quad M_{22} = a_{11}$$

Deleting the i-th row and j-th column

$$M_{11} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{22} \qquad M_{12} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{21}$$

$$M_{21} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{12} \qquad M_{22} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}$$

Minor of 2x2 Matrix

Example 13:

Compute the minor for matrix below.

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 5 \end{pmatrix}$$

Solution

$$M_{11} = \begin{vmatrix} 2 & -1 \\ 0 & 5 \end{vmatrix} = 5 \qquad M_{12} = \begin{vmatrix} 2 & -1 \\ 0 & 5 \end{vmatrix} = 0$$

$$M_{21} = \begin{vmatrix} 2 & -1 \\ 0 & 5 \end{vmatrix} = -1 \qquad M_{22} = \begin{vmatrix} 2 & -1 \\ 0 & 5 \end{vmatrix} = 2$$

$$\therefore \text{Minor } A = \begin{pmatrix} 5 & 0 \\ -1 & 2 \end{pmatrix}$$

COFACTOR

To find the cofactor of a matrix, just use the minors and apply the formula:

$$\text{Cofactor}, K = (-1)^{i+j} M_{ij}$$

Where M_{ij} is the minor in the i-th row, j-th column of the matrix.

Step to find a cofactor of 2x2 matrix.

1. Find the minor of the matrix.
2. Multiply each element of minor with (+) and (-) sign.

$$\text{Cofactor}, K = \begin{pmatrix} (+)M_{11} & (-)M_{12} \\ (-)M_{21} & (+)M_{22} \end{pmatrix}$$

Cofactor of 2x2 Matrix

Example 14:

Compute the cofactor for matrix below.

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 5 \end{pmatrix}$$

Solution

As calculation at Example 13, we get:

$$\text{Minor } A = \begin{pmatrix} 5 & 0 \\ -1 & 2 \end{pmatrix}$$

Therefore,

$$\text{Cofactor, } K = \begin{pmatrix} (+)5 & (-)0 \\ (-)-1 & (+)2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$$

ADJOINT

The adjoint of matrix A is the **transpose of cofactor** matrix of a given original matrix. The adjoint of matrix A is often written as adj(A).

$$\textbf{Adjoint A = adj(A)=}K^T$$

where K^T is cofactor of matrix A

Adjoint of 2x2 Matrix

Example 15:

Compute the adjoint for matrix below.

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 5 \end{pmatrix}$$

Solution

As calculation at Example 13, we get:

$$\text{Minor } A = \begin{pmatrix} 5 & 0 \\ -1 & 2 \end{pmatrix}$$

As calculation at Example 14, we get:

$$\text{Cofactor, } K = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$$

Therefore,

Adjoint A = adj(A)=K^T

$$\text{Adjoint } A = \text{adj}(A) = K^T = \begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix}$$

For Example!

Inverse of 2x2 Matrix

Example 16:

Compute the inverse matrix below.

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 5 \end{pmatrix}$$

Solution

Step 1: Find the determinant of matrix A, IAI

$$|A| = \begin{vmatrix} 2 & -1 \\ 0 & 5 \end{vmatrix}$$
$$= (2 \times 5) - (-1 \times 0)$$
$$= 10$$

Step to find an inverse matrix.
1. Find the determinant of the matrix.
2. Find the minor of the matrix.
3. Find the cofactor of the matrix.
4. Find the adjoint of the matrix by transpose the cofactor.
5. Find the inverse using:

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

Step 2: Find the minor.

$$\text{If } A = \begin{pmatrix} 2 & -1 \\ 0 & 5 \end{pmatrix}$$

Where : $M_{11} = 5 \quad M_{12} = 0$

$M_{21} = -1 \quad M_{22} = 2$

$$\text{Therefore, minor } A = \begin{pmatrix} 5 & 0 \\ -1 & 2 \end{pmatrix}$$

Step 3: Find the cofactor, K.

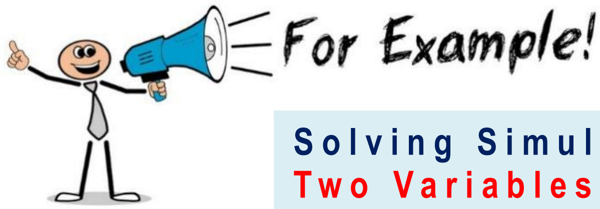
$$\text{Cofactor, } K = \begin{pmatrix} (+)5 & (-)0 \\ (-)-1 & (+)2 \end{pmatrix}$$
$$= \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$$

Step 4: Find the adjoint, adj(A).

$$\text{Adjoint } A = \text{adj}(A) = K^T = \begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix}$$

Step 5: Find the inverse matrix, A^{-1}.

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$
$$= \frac{1}{10} \times \begin{pmatrix} 5 & 1 \\ 0 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{5}{10} & \frac{1}{10} \\ \frac{0}{10} & \frac{2}{10} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{10} \\ 0 & \frac{1}{5} \end{pmatrix}$$



Solving Simultaneous Linear Equations Two Variables Using Inverse Method

Example 17:

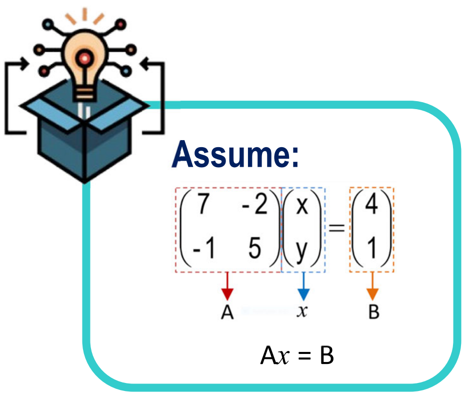
Solve the system of linear equations below using the inverse matrix method.

$$7x - 2y = 4$$
$$-x + 5y = 1$$

Solution

Change into matrix form.

$$\begin{pmatrix} 7 & -2 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \qquad \text{where } A = \begin{pmatrix} 7 & -2 \\ -1 & 5 \end{pmatrix}$$



Step 1: Find the determinant of matrix A, IAI

$$|A| = \begin{vmatrix} 7 & -2 \\ -1 & 5 \end{vmatrix}$$
$$= (7 \times 5) - ((-2) \times (-1))$$
$$= 35 - 2$$
$$= 33$$

Step 2: Find the minor.

$$\text{If } A = \begin{pmatrix} 7 & -2 \\ -1 & 5 \end{pmatrix}$$

Where : $M_{11} = 5 \quad M_{12} = -1$

$M_{21} = -2 \quad M_{22} = 7$

$$\text{Therefore, minor } A = \begin{pmatrix} 5 & -1 \\ -2 & 7 \end{pmatrix}$$

Step 3: Find the cofactor, K.

$$\text{Cofactor, } K = \begin{pmatrix} (+)5 & (-)-1 \\ (-)-2 & (+)7 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 1 \\ 2 & 7 \end{pmatrix}$$

Step 4: Find the adjoint, adj(A).

$$\text{Adjoint } A = \text{adj}(A) = K^T = \begin{pmatrix} 5 & 2 \\ 1 & 7 \end{pmatrix}$$

Step 5: Find the inverse matrix, A^{-1}.

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

$$= \frac{1}{33} x \begin{pmatrix} 5 & 2 \\ 1 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{33} & \frac{2}{33} \\ \frac{1}{33} & \frac{7}{33} \end{pmatrix}$$

Step 6: Find x=A⁻¹B.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{5}{33} & \frac{2}{33} \\ \frac{1}{33} & \frac{7}{33} \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{33}(4) + \frac{2}{33}(1) \\ \frac{1}{33}(4) + \frac{7}{33}(1) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$$

$$\therefore x = \frac{2}{3},\ y = \frac{1}{3}$$

Assume:

$$\begin{pmatrix} 7 & -2 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

A x B

Ax = B

Therefore, x = A^{-1} B

For 3x3 matrix:

MINOR

If B is a square matrix, the minor of the element is the determinant of the sub matrix formed by deleting the i-th row and j-th column. This number is often denoted M_{ij}.

$$\text{Let } B = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Therefore,

$$\text{Minor } B = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$$

Deleting the i-th row and j-th column.

$$M_{11} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32}$$

$$M_{12} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = a_{21}a_{33} - a_{23}a_{31}$$

$$M_{13} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{21}a_{32} - a_{22}a_{31}$$

$$M_{21} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = a_{12}a_{33} - a_{13}a_{32}$$

$$M_{22} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = a_{11}a_{33} - a_{13}a_{31}$$

$$M_{23} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} = a_{11}a_{32} - a_{12}a_{31}$$

$$M_{31} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} = a_{12}a_{23} - a_{13}a_{22}$$

$$M_{32} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = a_{11}a_{23} - a_{13}a_{21}$$

$$M_{33} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Minor of 3x3 Matrix

Example 18:

Find the minor for matrix $A = \begin{pmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{pmatrix}$

$$M_{11} = \begin{vmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = (3 \times 5) - (2 \times 4) = 7$$

$$M_{12} = \begin{vmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = (1 \times 5) - (2 \times 2) = 1$$

$$M_{13} = \begin{vmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = (1 \times 4) - (3 \times 2) = -2$$

$$M_{21} = \begin{vmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{vmatrix} = \begin{vmatrix} -2 & 2 \\ 4 & 5 \end{vmatrix} = (-2 \times 5) - (2 \times 4) = -18$$

$$M_{22} = \begin{vmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{vmatrix} = \begin{vmatrix} -3 & 2 \\ 2 & 5 \end{vmatrix} = (-3 \times 5) - (2 \times 2) = -19$$

$$M_{23} = \begin{vmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{vmatrix} = \begin{vmatrix} -3 & -2 \\ 2 & 4 \end{vmatrix} = (-3 \times 4) - (-2 \times 2) = -8$$

$$M_{31} = \begin{vmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{vmatrix} = \begin{vmatrix} -2 & 2 \\ 3 & 2 \end{vmatrix} = (-2 \times 2) - (2 \times 3) = -10$$

$$M_{32} = \begin{vmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{vmatrix} = \begin{vmatrix} -3 & 2 \\ 1 & 2 \end{vmatrix} = (-3 \times 2) - (2 \times 1) = -8$$

$$M_{33} = \begin{vmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{vmatrix} = \begin{vmatrix} -3 & -2 \\ 1 & 3 \end{vmatrix} = (-3 \times 3) - (-2 \times 1) = -7$$

Solution

$$\text{Therefore, minor } A = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$$

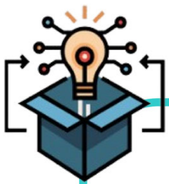
$$= \begin{pmatrix} 7 & 1 & -2 \\ -18 & -19 & -8 \\ -10 & -8 & -7 \end{pmatrix}$$

COFACTOR

To find the cofactor of a matrix, just use the minors and apply the formula:

$$\text{Cofactor}, K = (-1)^{i+j} M_{ij}$$

Where M_{ij} is the minor in the i-th row, j-th column of the matrix.



Step to find a cofactor of 3x3 matrix.

1. Find the minor of the matrix.
2. Multiply each element of minor with (+) and (-) sign.

$$\text{Cofactor}, K = \begin{pmatrix} (+)M_{11} & (-)M_{12} & (+)M_{13} \\ (-)M_{21} & (+)M_{22} & (-)M_{23} \\ (+)M_{31} & (-)M_{32} & (+)M_{33} \end{pmatrix}$$

Cofactor of 3x3 Matrix

Example 19:

Find the cofactor for matrix $A = \begin{pmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{pmatrix}$

Solution

As calculation at Example 18, we get:

$$\text{Minor } A = \begin{pmatrix} 7 & 1 & -2 \\ -18 & -19 & -8 \\ -10 & -8 & -7 \end{pmatrix}$$

Therefore,

$$\text{Cofactor, } K = \begin{pmatrix} (+)7 & (-)1 & (+)-2 \\ (-)-18 & (+)-19 & (-)-8 \\ (+)-10 & (-)-8 & (+)-7 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -1 & -2 \\ 18 & -19 & 8 \\ -10 & 8 & -7 \end{pmatrix}$$

ADJOINT

The adjoint of matrix A is the **transpose of cofactor** matrix of a given original matrix. The adjoint of matrix A is often written as adj(A).

Adjoint A = adj(A)=K^T

where K^T is cofactor of matrix A.

Adjoint of 3x3 Matrix

Example 20:

Find the adjoint for matrix $A = \begin{pmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{pmatrix}$

Solution

As calculation at Example 18, we get:

$$\text{Minor } A = \begin{pmatrix} 7 & 1 & -2 \\ -18 & -19 & -8 \\ -10 & -8 & -7 \end{pmatrix}$$

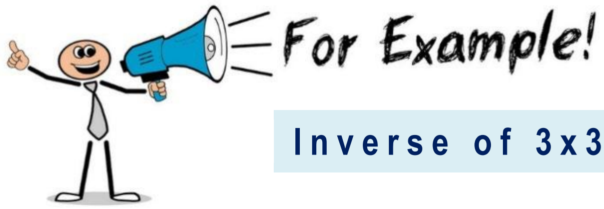
As calculation at Example 19, we get:

$$\text{Cofactor, } K = \begin{pmatrix} 7 & -1 & -2 \\ 18 & -19 & 8 \\ -10 & 8 & -7 \end{pmatrix}$$

Therefore,

Adjoint A = adj(A)=K^T

$$\text{Adjoint } A = \text{adj}(A) = K^T = \begin{pmatrix} 7 & 18 & -10 \\ -1 & -19 & 8 \\ -2 & 8 & -7 \end{pmatrix}$$



Inverse of 3x3 Matrix

Example 21:

Find the inverse matrix below.

$$A = \begin{pmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{pmatrix}$$

Step to find an inverse matrix.

1. Find the determinant of the matrix.
2. Find the minor of the matrix.
3. Find the cofactor of the matrix.
4. Find the adjoint of the matrix by transpose the cofactor.
5. Find the inverse using:

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

Solution

Step 1: Find the determinant of matrix A, IAI

By using cofactor expansion:

$$|A| = -3\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} - (-2)\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} + 2\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$$

$$= -3\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} + 2\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$$

$$= -3[(3\times 5)-(2\times 4)] + 2[(1\times 5)-(2\times 2)] + 2[(1\times 4)-(3\times 2)]$$

$$= -3(15-8) + 2(5-4) + 2(4-6)$$

$$= -3(7) + 2(1) + 2(-2)$$

$$= -21 + 2 - 4$$

$$= -23$$

Or by using cross multiplication

$$\therefore |A| = \left(\begin{array}{ccc|cc} -3 & -2 & 2 & -3 & -2 \\ 1 & 3 & 2 & 1 & 3 \\ 2 & 4 & 5 & 2 & 4 \end{array}\right)$$

=[8+(-8)+(-45)] – [12+(-24)+(-10)]

= -45-(-22)

= -23

Step 2: Find the minor.

$$A = \begin{pmatrix} -3 & -2 & 2 \\ 1 & 3 & 2 \\ 2 & 4 & 5 \end{pmatrix}$$

$$M_{11} = \begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = (3\times5)-(2\times4) = 7$$

$$M_{12} = \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = (1\times5)-(2\times2) = 1$$

$$M_{13} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = (1\times4)-(3\times2) = -2$$

$$M_{21} = \begin{vmatrix} -2 & 2 \\ 4 & 5 \end{vmatrix} = (-2\times5)-(2\times4) = -18$$

$$M_{22} = \begin{vmatrix} -3 & 2 \\ 2 & 5 \end{vmatrix} = (-3\times5)-(2\times2) = -19$$

$$M_{23} = \begin{vmatrix} -3 & -2 \\ 2 & 4 \end{vmatrix} = (-3\times4)-(-2\times2) = -8$$

$$M_{31} = \begin{vmatrix} -2 & 2 \\ 3 & 2 \end{vmatrix} = (-2\times2)-(2\times3) = -10$$

$$M_{32} = \begin{vmatrix} -3 & 2 \\ 1 & 2 \end{vmatrix} = (-3\times2)-(2\times1) = -8$$

$$M_{33} = \begin{vmatrix} -3 & -2 \\ 1 & 3 \end{vmatrix} = (-3\times3)-(-2\times1) = -7$$

$$\text{Therefore, Minor } A = \begin{pmatrix} 7 & 1 & -2 \\ -18 & -19 & -8 \\ -10 & -8 & -7 \end{pmatrix}$$

Step 3: Find the cofactor, K.

$$\text{Cofactor, } K = \begin{pmatrix} (+)7 & (-)1 & (+)-2 \\ (-)-18 & (+)-19 & (-)-8 \\ (+)-10 & (-)-8 & (+)-7 \end{pmatrix} = \begin{pmatrix} 7 & -1 & -2 \\ 18 & -19 & 8 \\ -10 & 8 & -7 \end{pmatrix}$$

Step 4: Find the adjoint, adj(A).

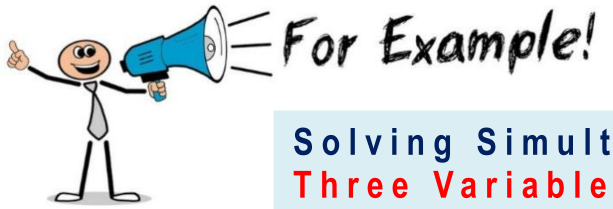
$$\text{Adjoint A} = \text{adj(A)} = K^T = \begin{pmatrix} 7 & 18 & -10 \\ -1 & -19 & 8 \\ -2 & 8 & -7 \end{pmatrix}$$

Step 5: Find the inverse matrix, A^{-1}.

$$A^{-1} = \frac{1}{|A|}\text{adj(A)}$$

$$= \frac{1}{-23} \times \begin{pmatrix} 7 & 18 & -10 \\ -1 & -19 & 8 \\ -2 & 8 & -7 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{7}{23} & -\frac{18}{23} & \frac{10}{23} \\ \frac{1}{23} & \frac{19}{23} & -\frac{8}{23} \\ \frac{2}{23} & -\frac{8}{23} & \frac{7}{23} \end{pmatrix}$$



Solving Simultaneous Linear Equations Three Variables Using Inverse Method

Example 22:

Solve the system of linear equations below using the inverse matrix method.

$$3x + 2y + 4z = 3$$
$$x + y + z = 2$$
$$2x - y + 3z = -3$$

Solution

Change into matrix form.

$$\begin{pmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} \qquad \text{where } A = \begin{pmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{pmatrix}$$

Assume:

$$\underbrace{\begin{pmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{pmatrix}}_{A}\underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}}_{B}$$

$Ax = B$

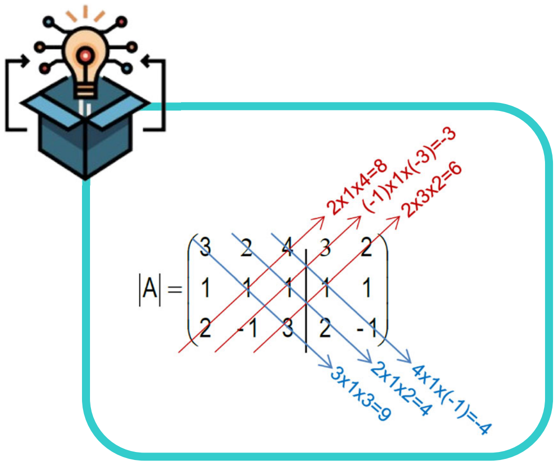
Step 1: Find the determinant of matrix A, IAI

By using cofactor expansion:

$$|A| = 3\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + 4\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$$
$$= 3[(1\times 3) - (1\times(-1))] - 2[(1\times 3) - (1\times 2)] + 4[(1\times(-1)) - (1\times 2)]$$
$$= 3(3+1) - 2(3-2) + 4(-1-2)$$
$$= 3(4) - 2(1) + 4(-3)$$
$$= 12 - 2 - 12$$
$$= -2$$

Or by using cross multiplication

$$\therefore |A| = \left(\begin{array}{ccc|cc} 3 & 2 & 4 & 3 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & -1 & 3 & 2 & -1 \end{array}\right)$$



$=[(4\times1\times(-1))+(2\times1\times2)+(3\times1\times3)]-[(2\times1\times4)+((-1)\times1\times(-3))+(2\times3\times2)]$

$=(-4+4+9)-(28+(-3)+6)$

$= 9-11$

$= -2$

Step 2: Find the minor.

$$\text{If } A = \begin{pmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{pmatrix}$$

$$M_{11} = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = (1\times 3) - (1\times(-1) = 4$$

$$M_{12} = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = (1\times 3) - (1\times 2) = 1$$

$$M_{13} = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = (1\times(-1)) - (1\times 2) = -3$$

$$M_{21} = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 4 \\ -1 & 3 \end{vmatrix} = (2\times 3) - (4\times(-1)) = 10$$

$$M_{22} = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = (3\times 3) - (4x2) = 1$$

$$M_{23} = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = (3\times(-1)) - (2x2) = -7$$

$$M_{31} = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} = (2\times 1) - (4\times 1) = -2$$

$$M_{32} = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} = (3\times 1) - (4\times 1) = -1$$

$$M_{33} = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = (3\times 1) - (2\times 1) = 1$$

$$\text{Therefore, minor } A = \begin{pmatrix} 4 & 1 & -3 \\ 10 & 1 & -7 \\ -2 & -1 & 1 \end{pmatrix}$$

Step 3: Find the cofactor, K.

$$\text{Cofactor, } K = \begin{pmatrix} (+)4 & (-)1 & (+)-3 \\ (-)10 & (+)1 & (-)-7 \\ (+)-2 & (-)-1 & (+)1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -1 & -3 \\ -10 & 1 & 7 \\ -2 & 1 & 1 \end{pmatrix}$$

Step 4: Find the adjoint, adj(A).

$$\text{Adjoint } A = \text{adj}(A) = K^T = \begin{pmatrix} 4 & -10 & -2 \\ -1 & 1 & 1 \\ -3 & 7 & 1 \end{pmatrix}$$

Step 5: Find the inverse matrix, A^{-1}.

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

$$= \frac{1}{-2} \times \begin{pmatrix} 4 & -10 & -2 \\ -1 & 1 & 1 \\ -3 & 7 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 5 & 1 \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{7}{2} & -\frac{1}{2} \end{pmatrix}$$

Step 6: Find x=A^{-1}B.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 & 5 & 1 \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{7}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} -2(3) + 5(2) + 1(-3) \\ \frac{1}{2}(3) + \left(-\frac{1}{2}\right)(2) + \left(-\frac{1}{2}\right)(-3) \\ \left(\frac{3}{2}\right)(3) + \left(-\frac{7}{2}\right)(2) + \left(-\frac{1}{2}\right)(-3) \end{pmatrix}$$

$$= \begin{pmatrix} -2(3) + 5(2) + 1(-3) \\ \frac{1}{2}(3) - \left(\frac{1}{2}\right)(2) - \left(\frac{1}{2}\right)(-3) \\ \left(\frac{3}{2}\right)(3) - \left(\frac{7}{2}\right)(2) - \left(\frac{1}{2}\right)(-3) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$\therefore x = 1,\ y = 2,\ z = -1$$

Assume:

$$\underbrace{\begin{pmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}}_{B}$$

Ax = B

Therefore, $\boldsymbol{x}$ = **A^{-1} B**

1.3.2 Cramer's Rule

Cramer's rule can be used in a linear system involving two and three variables. We need skill to find the determinant of matrices.

For 2x2 matrix:

Consider the following linear system:

$$a_1x + b_1y = d_1$$
$$a_2x + b_2y = d_2$$

Step 1: Change into matrix form.

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \qquad \text{where } A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$$

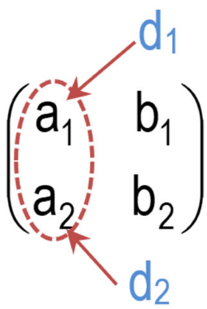
Step 2: Find the determinant of matrix A, IAI.

Step 3: Find the **determinant of x-matrix**, **IA_xI**

The x-matrix can be obtained by:

substitute column $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ with $\begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$ on matrix A

$$\text{x-matrix}: A_x = \begin{pmatrix} d_1 & b_1 \\ d_2 & b_2 \end{pmatrix}$$

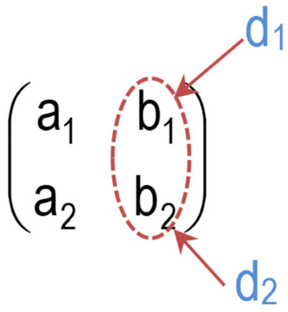


Step 4: Find the **determinant of y-matrix**, **$|A_y|$**

The x-matrix can be obtained by:

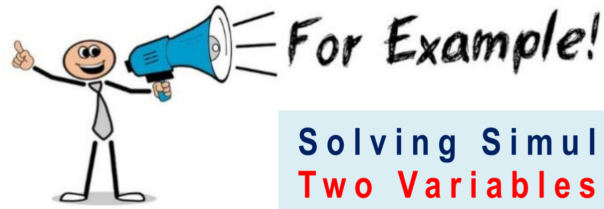
substitute column $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ with $\begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$ on matrix A

$$\text{y - matrix} : A_y = \begin{pmatrix} a_1 & d_1 \\ a_2 & d_2 \end{pmatrix}$$



Step 5: Solve the equations for x and y.

$$x = \frac{|A_x|}{|A|} \qquad y = \frac{|A_y|}{|A|}$$



Solving Simultaneous Linear Equations Two Variables Using Cramer's Rules

Example 23:

Solve the system of linear equations below using Cramer's rule.

$$3x - 2y = -16$$
$$7x - 5y = -39$$

Solution

Step 1: Change into matrix form.

$$\begin{pmatrix} 3 & -2 \\ 7 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -16 \\ -39 \end{pmatrix}$$

Step 2: Find the determinant of matrix A, IAI

$$A = \begin{pmatrix} 3 & -2 \\ 7 & -5 \end{pmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 3 & -2 \\ 7 & -5 \end{vmatrix} \\ &= (3 \times (-5)) - ((-2) \times 7) \\ &= -1 \end{aligned}$$

Step 3: Find the **determinant of x-matrix**, $|A_x|$

$$A_x = \begin{pmatrix} -16 & -2 \\ -39 & -5 \end{pmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} -16 & -2 \\ -39 & -5 \end{vmatrix} \\ &= [(-16)\times(-5) - (-2)\times(-39)] \\ &= 2 \end{aligned}$$

Step 4: Find the **determinant of y-matrix**, $|A_y|$

$$A_y = \begin{pmatrix} 3 & -16 \\ 7 & -39 \end{pmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 3 & -16 \\ 7 & -39 \end{vmatrix} \\ &= [3\times(-39) - (-16)\times(7)] \\ &= -5 \end{aligned}$$

Step 5: Solve the equations for x and y.

$$x = \frac{|A_x|}{|A|} = \frac{2}{-1}, \quad x = -2 \qquad\qquad y = \frac{|A_y|}{|A|} = \frac{-5}{-1}, \quad y = 5$$

For 3x3 matrix:

Consider the following linear system:

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Step 1: Change into matrix form.

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} \quad \text{where } A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

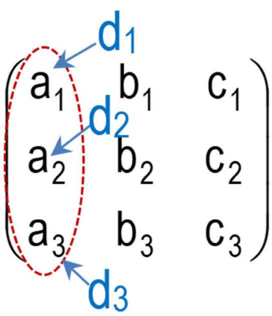
Step 2: Find the determinant of matrix A, IAI.

Step 3: Find the determinant of **x-matrix**, IA_xI

The x-matrix can be obtained by:

substitute column $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ with $\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ on matrix A

$$x\text{-matrix}: A_x = \begin{pmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{pmatrix}$$

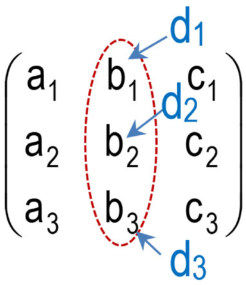


Step 4: Find the determinant of **y-matrix**, IA_yI

The x-matrix can be obtained by:

substitute column $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ with $\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ on matrix A

$$\text{y - matrix} : A_y = \begin{pmatrix} a_1 & d_1 & c_1 \\ b_2 & d_2 & c_2 \\ c_3 & d_3 & c_3 \end{pmatrix}$$

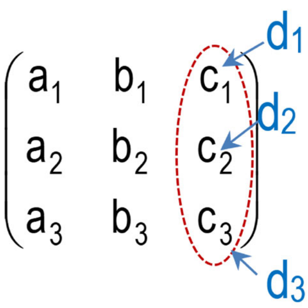


Step 5: Find the determinant of **z-matrix**, IA_zI

The x-matrix can be obtained by:

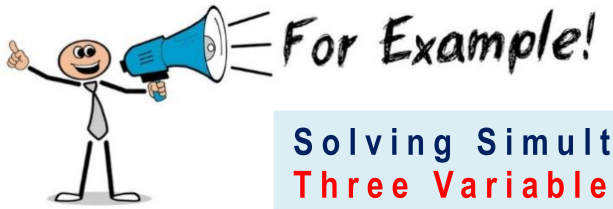
substitute column $\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$ with $\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ on matrix A

$$\text{z - matrix} : A_z = \begin{pmatrix} a_1 & b_1 & d_1 \\ b_2 & b_2 & d_2 \\ c_3 & b_3 & d_3 \end{pmatrix}$$



Step 6: Solve the equations for x, y and z.

$$x = \frac{|A_x|}{|A|} \qquad y = \frac{|A_y|}{|A|} \qquad z = \frac{|A_z|}{|A|}$$



Solving Simultaneous Linear Equations Three Variables Using Cramer's Rules

Example 24:

Solve the system of linear equations below using Cramer's rule.

$$5x - y + 7z = 4$$
$$6x - 2y + 9z = 5$$
$$2x + 8y - 4z = 8$$

Solution

Step 1: Change into matrix form.

$$\begin{pmatrix} 5 & -1 & 7 \\ 6 & -2 & 9 \\ 2 & 8 & -4 \end{pmatrix}\begin{pmatrix} x \\ 7 \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix}$$

Step 2: Find the determinant of matrix A, IAI

$$A = \begin{pmatrix} 5 & -1 & 7 \\ 6 & -2 & 9 \\ 2 & 8 & -4 \end{pmatrix}$$

$$|A| = 5\begin{vmatrix} -2 & 9 \\ 8 & -4 \end{vmatrix} - (-1)\begin{vmatrix} 6 & 9 \\ 2 & -4 \end{vmatrix} + 7\begin{vmatrix} 6 & -2 \\ 2 & 8 \end{vmatrix}$$

$$= 5\begin{vmatrix} -2 & 9 \\ 8 & -4 \end{vmatrix} + 1\begin{vmatrix} 6 & 9 \\ 2 & -4 \end{vmatrix} + 7\begin{vmatrix} 6 & -2 \\ 2 & 8 \end{vmatrix}$$

$$= 5[(-2 \times -4) - (9 \times 8)] + 1[(6 \times -4) - (9 \times 2)] + 7[(6 \times 8) - (-2 \times 2)]$$

$$= 2$$

Step 3: Find the determinant of x-matrix, $|A_x|$

$$A_x = \begin{pmatrix} 4 & -1 & 7 \\ 5 & -2 & 9 \\ 8 & 8 & -4 \end{pmatrix}$$

$$\begin{aligned} |A_x| &= 4\begin{vmatrix} -2 & 9 \\ 8 & -4 \end{vmatrix} - (-1)\begin{vmatrix} 5 & 9 \\ 8 & -4 \end{vmatrix} + 7\begin{vmatrix} 5 & -2 \\ 8 & 8 \end{vmatrix} \\ &= 4\begin{vmatrix} -2 & 9 \\ 8 & -4 \end{vmatrix} + 1\begin{vmatrix} 5 & 9 \\ 8 & -4 \end{vmatrix} + 7\begin{vmatrix} 5 & -2 \\ 8 & 8 \end{vmatrix} \\ &= 4[(-2 \times -4) - (9 \times 8)] + 1[(5 \times -4) - (9 \times 8)] + 7[(5 \times 8) - (-2 \times 8)] \\ &= 44 \end{aligned}$$

Step 4: Find the determinant of y-matrix, $|A_y|$

$$A_y = \begin{pmatrix} 5 & 4 & 7 \\ 6 & 5 & 9 \\ 2 & 8 & -4 \end{pmatrix}$$

$$\begin{aligned} |A_y| &= 5\begin{vmatrix} 5 & 9 \\ 8 & -4 \end{vmatrix} - 4\begin{vmatrix} 6 & 9 \\ 2 & -4 \end{vmatrix} + 7\begin{vmatrix} 6 & 5 \\ 2 & 8 \end{vmatrix} \\ &= 5[(5 \times -4) - (9 \times 8)] - 4[(6 \times -4) - (9 \times 2)] + 7[(6 \times 8) - (5 \times 2)] \\ &= -26 \end{aligned}$$

Step 5: Find the determinant of z-matrix, $|A_z|$

$$A_z = \begin{pmatrix} 5 & -1 & 4 \\ 6 & -2 & 5 \\ 2 & 8 & 8 \end{pmatrix}$$

$$|A_z| = 5\begin{vmatrix} -2 & 5 \\ 8 & 8 \end{vmatrix} - (-1)\begin{vmatrix} 6 & 5 \\ 2 & 8 \end{vmatrix} + 4\begin{vmatrix} 6 & -2 \\ 2 & 8 \end{vmatrix}$$

$$= 5\begin{vmatrix} -2 & 5 \\ 8 & 8 \end{vmatrix} + 1\begin{vmatrix} 6 & 5 \\ 2 & 8 \end{vmatrix} + 4\begin{vmatrix} 6 & -2 \\ 2 & 8 \end{vmatrix}$$

$$= 5[(-2\times 8) - (5\times 8)] + 1[(6\times 8) - (5\times 2)] + 4[(6\times 8) - (-2\times 2)]$$

$$= -34$$

Step 6: Solve the equations for x, y and z.

$$x = \frac{|A_x|}{|A|} = \frac{44}{2}, \quad x = 22$$

$$y = \frac{|A_y|}{|A|} = \frac{-26}{2}, \quad y = -13$$

$$z = \frac{|A_y|}{|A|} = \frac{-34}{2}, \quad z = -17$$

MATH DRILLS!

Answer all the questions.

EXERCISES 6

Solve the system of linear equations below by using the inverse matrix method and Cramer's rule:

a)

$$x + 3y + 3z = 4$$
$$2x - 3y - 2z = 5$$
$$3x + y + 2z = 5$$

b)

$$2x - y + 2z = -3$$
$$6x - 2y + z = -2$$
$$-3x + 4y - 3z = 6$$

c)

$$4a - 5b + 6c = 3$$
$$8a - 7b - 3c = 9$$
$$-2a + b + c = 1$$

d)

$$2x + 2y + 2z = 1$$
$$2x - 2y + 3z = 10$$
$$-x + 2y - z = 7$$

HOMEWORK

FINAL EXAMINATION QUESTION SAMPLE

HOMEWORK

1. Given matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 & -1 \\ 0 & 3 & 2 \end{pmatrix}$

a) Express A^T
b) Express B^T
c) Calculate $A+B^T$
d) Calculate $A^T - B$
e) Calculate AB

2. If $A = \begin{pmatrix} -2 & -1 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}$

a) AB
b) BA

3. If $M = \begin{pmatrix} 3 & 5 & -3 \\ 0 & 8 & 4 \\ -1 & 7 & 2 \end{pmatrix}$ and $N = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$, calculate MN.

4. Given matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ -1 & 3 \\ -1 & 2 \end{pmatrix}$, calculate :

a) A +B
b) A - B

5. If $A = \begin{pmatrix} 1 & 2 \\ 5 & 3 \\ 7 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 5 & -3 \\ 0 & 8 & 4 \\ 1 & 7 & 2 \end{pmatrix}$, calculate :

a) BA
b) B^2

6. Calculate the determinants for the following :

a) $P = \begin{pmatrix} -7 & -1 \\ -1 & 3 \end{pmatrix}$

b) $C = \begin{pmatrix} 0 & -1 & 1 \\ 2 & 2 & 1 \\ 1 & -3 & 3 \end{pmatrix}$

7. Solve the system of linear equations below by using the inverse matrix method and Cramer's rule :

$$4x + 8y + 3z = 2$$
$$3x + 5y + z = -3$$
$$x + 4y + 3z = 5$$

-End of Homework-

FINAL EXAMINATION QUESTION SAMPLE

1(a) Referring to matrix $R = \begin{pmatrix} 1 & 2 & 4 \\ 4 & 8 & 7 \\ 5 & 9 & 5 \end{pmatrix}$, identify :

i. The elements of R_{31} and R_{12}

ii. R^T

1(b) i. Given that $X = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 7 & 3 \end{pmatrix}$, $Y = \begin{pmatrix} 4 & -2 & 1 \\ 4 & 2 & 3 \end{pmatrix}$ and $Z = \begin{pmatrix} 1 & 6 & 5 \\ 0 & -2 & 9 \end{pmatrix}$.

Calculate $4X - 3Y + Z$.

ii. Calculate the value of p, q and r in the following matrix equation :

$$3\begin{pmatrix} p & 1 \\ 2 & 5 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 4 & q \\ 1 & 2 \\ 0 & q \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 5 & 13 \\ 9 & 5 \end{pmatrix}$$

1(c) Solve the following equations using cramer's rule..

$$x + 3y + 2z = 3$$
$$2x - y - 3z = -8$$
$$5x + 2y + z = 9$$

-End of Final Examination Question Sample-

ANSWERS: EXERCISES

EXERCISE 1

1. a) $B_{11} = 1$ b) $B_{13} = 2$ c) $B_{22} = 5$
2. a) $C_{12} = 3$ b) $B_{21} = 0$ c) $B_{32} = 6$

EXERCISE 2

a) 1×2 matrix

b) 3×1 matrix

c) 3×3 matrix

d) 3×2 matrix

EXERCISE 3

1 a) Cannot be done because the order of matrices are not the same.

b) $\begin{pmatrix} 1 & 2 & -1 \\ 4 & 1 & 6 \end{pmatrix}$

c) $\begin{pmatrix} 3 & -2 & -7 \\ 4 & -7 & -8 \end{pmatrix}$

d) $\begin{pmatrix} 3 & 1 \\ 1 & -3 \\ 1 & -5 \end{pmatrix}$

2 a) $\begin{pmatrix} -2 & 0 \\ 7 & 6 \end{pmatrix}$

b) $\begin{pmatrix} 6 & -2 \\ 1 & -6 \end{pmatrix}$

c) $\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 6 & 1 \\ -2 & -6 \end{pmatrix}$

EXERCISE 4

i) $\begin{pmatrix} -27 & -15 \\ 24 & 36 \end{pmatrix}$

ii) $\begin{pmatrix} 30 & -18 \\ 1 & -21 \end{pmatrix}$

EXERCISE 5

a) - 22 b) 23 c) - 22 b) - 8

EXERCISE 6

a) $x = 16,\ y = 35,\ z = -39$

b) $x = \frac{2}{25},\ y = \frac{3}{5},\ z = -\frac{32}{25}$

c) $x = -\frac{24}{7},\ y = -\frac{33}{7},\ z = -\frac{8}{7}$

d) $x = -21,\ y = \frac{5}{2},\ z = 19$

ANSWERS: HOMEWORK

1 a) $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 5 & 7 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 0 \\ -1 & 3 \\ -1 & 2 \end{pmatrix}$

c) $\begin{pmatrix} 3 & 3 \\ 1 & 8 \\ -1 & 9 \end{pmatrix}$ d) $\begin{pmatrix} -1 & 3 & 1 \\ 3 & 2 & 5 \end{pmatrix}$

e) $\begin{pmatrix} 2 & 8 & 5 \\ 4 & 13 & 8 \\ 0 & 21 & 4 \end{pmatrix}$

2 a) $\begin{pmatrix} -2 & -1 \\ 2 & -1 \end{pmatrix}$ b) $\begin{pmatrix} -4 & -1 \\ 8 & 1 \end{pmatrix}$

3 a) $\begin{pmatrix} -4 \\ 28 \\ 15 \end{pmatrix}$

4 a) $\begin{pmatrix} 3 & 3 \\ 1 & 8 \\ -1 & 9 \end{pmatrix}$ b) $\begin{pmatrix} -1 & 3 \\ 3 & 2 \\ 1 & 5 \end{pmatrix}$

5 a) $\begin{pmatrix} 23 & -6 \\ 76 & 60 \\ 58 & 37 \end{pmatrix}$ b) $\begin{pmatrix} 12 & 34 & 5 \\ -4 & 92 & 40 \\ -5 & 65 & 35 \end{pmatrix}$

6 a) -22 b) -3

7 $x = 23$
$y = -18$
$z = 18$

ANSWERS: FINAL EXAMINATION QUESTION SAMPLE

1(a) i. $R_{31} = 5,\ R_{12} = 2$

ii. $R^T = \begin{pmatrix} 1 & 4 & 5 \\ 2 & 8 & 9 \\ 4 & 7 & 5 \end{pmatrix}$

1(b) i. $\begin{pmatrix} -7 & 28 & 14 \\ 5 & 20 & 12 \end{pmatrix}$.

ii. $p = \frac{7}{3},\ q = -1,\ r = 7$

1(c) $x = 2$

$y = -3$

$z = 5$

REFERENCES

Nur Farhana Sharip, Nurulain Ahmad, Nur Elyani Musa & Haryani Hamid (2007). *Engineering Mathematics 1.* Selangor: Oxford Fajar Bakti Sdn. Bhd.

Stewart, J., Redlin, L. & Watson, S. (2006). Precalculus: Mathematics for Calculus (5th Edition). Boston. Thompson Learning, Inc.

ENGINEERING MATHEMATICS 1 :
MATRICES

FOR MALAYSIAN POLYTECHNIC STUDENTS

"This book is one of the mathematical references that covers a solid understanding of the matrices. In this book, we will explore the fundamentals of matrices, types of matrices, operations on matrices and solving simultaneous linear equations up to three variables using matrices. This book also includes easy-to-understand explanations and exam-shaped questions."

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